Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Statistics 4 (6686/01)

ALWAYS LEARNING

PEARSON

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2015
Publications Code UA042714
All the material in this publication is copyright
© Pearson Education Ltd 2015

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\sqsubseteq$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a

misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

June 2015
6686 S4
Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1. (a)</td><td>

Store	A	B	C	D	E	F	G	H
Difference *July-Jan*	33	63	121	–60	–54	24	–19	33

</td><td>B1</td></tr>
<tr><td></td><td>$\bar{d} = \frac{141}{8} = (\pm)17.625$</td><td>M1</td></tr>
<tr><td></td><td>$s_d^{\ 2} = \frac{8}{7}\left(\frac{28241}{8} - 17.625^2\right) = 3679.4\ldots$
or
$s_d^{\ 2} = \frac{1}{7}\left(28241 - \frac{141^2}{8}\right) = 3679.4\ldots$</td><td>M1</td></tr>
<tr><td></td><td>To test $\mathrm{H_0} : \mu_d = 0$ against $\mathrm{H_1} : \mu_d > 0$ (o.e.)</td><td>B1</td></tr>
<tr><td></td><td>Test stat
$t = \frac{17.625 - 0}{\sqrt{\frac{3679.4\ldots}{8}}} = 0.8218\ldots$</td><td>M1A1cso</td></tr>
<tr><td></td><td>Critical value, $t_7 = 1.895$</td><td>B1</td></tr>
<tr><td></td><td>Not in critical region therefore insufficient reason to reject $\mathrm{H_0}$
No significant evidence that on average stores sell more lottery tickets in July than in January</td><td>A1ft
(8)</td></tr>
<tr><td>(b)</td><td>Need assumption that the underlying distribution of the difference in sales in July and in January is normally distributed.</td><td>B1 (1)</td></tr>
<tr><td></td><td></td><td>Total 9</td></tr>
<tr><td></td><td>Notes</td><td></td></tr>
<tr><td>(a)</td><td colspan="2">1st B1 for differences all correct (o.e.)

1st M1 attempt to find $\bar{d} = \frac{\sum \text{"their } d\text{"}}{8}$

2nd M1 attempting s_d or $s_d^{\ 2}$ $\frac{1}{7}\left(\sum \text{"their } d^2\text{"} - \frac{\left(\sum \text{"their } d\text{"}\right)^2}{8}\right)$

2nd B1 both correct in terms of μ or μ_d (allow a defined symbol) condone $\mu_{July-Jan}$

3rd M1 for attempting the correct test statistic $\frac{\bar{d}}{s_d/\sqrt{8}}$

1st A1cso awrt 0.822 with no errors.

3rd B1 alternate method, p value of 0.219. Allow 2.365 for 2-tail test

Final A1 need conclusion in context, need tickets July and January, ft their test stat and critical value

NB difference of 2 means test gains no marks</td></tr>
<tr><td>(b)</td><td colspan="2">B1 need differences to be normally distributed, not just normal distribution</td></tr>
</table>

Question Number	Scheme	Marks
2. (a)	$n=8 \quad \sum x = 843 \quad \sum x^2 = 89211$ $\therefore \quad \bar{x} = 105.375$ $s^2 = \frac{8}{7}\left(\frac{89211}{8} - 105.375^2\right) = 54.2678\ldots$ or $s^2 = \frac{1}{7}\left(89211 - \frac{843^2}{8}\right) = 54.2678\ldots$	M1A1
	Confidence interval is given by $\frac{7 \times 54.267\ldots}{14.067} < \sigma^2 < \frac{7 \times 54.267\ldots}{2.167}$	M1B1
	$\therefore \; 27.004\ldots < \sigma^2 < 175.299\ldots$ $5.1966\ldots < \sigma < 13.240\ldots$	M1d A1 (6)
(b)	Need to assume underlying **Normal** distribution for **weights** of blocks of cheese.	B1 (1)
(c)	Lower limit of CI is $> 5\ g$ suggests that **Fred** needs **training**.	B1ft (1)
(d)	To test $\quad H_0 : \mu = 100\ , \quad H_1 : \mu \neq 100 \quad (\mu > 100)$ where μ is the mean weight of blocks of cheese	B1
	Test statistic $\quad t = \frac{102.6 - 100}{\sqrt{\frac{19.4}{20}}} = 2.6399\ldots$	M1A1
	Critical value(s): $\quad t_{19} = (\pm)1.729\ (1.328)$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1	A1ft
	Significant evidence that the **mean weight** of the **blocks** of cheese is not 100 g (more than 100g)	B1cso (6)
		Total 14
	Notes	
(a)	1st M1 attempting s or s^2 1st A1 awrt 54.3 2nd M1 for $\frac{7s^2}{\chi^2}$ B1 14.067 & 2.167 3rd M1d Dept on previous M mark. Rearranging leading to interval for σ- must square root A1 awrt 5.20 and 13.2 (allow 5.2) **NB** a correct interval gains full marks	
(c)	B1ft on their CI must have **Fred/He/employee (do not allow empoloyees)** and **training**. They must have an interval in part(a)	
(d)	1st B1 Both hypotheses with μ. Allow one-tail 1st M1 $\frac{102.6-100}{\frac{s \text{ or } s^2}{\sqrt{20}}}$ 1st A1 awrt 2.64 2nd B1 allow p value of 0.0161 in place of critical value. CV must follow from H_1 2nd A1ft a correct statement – do not allow contradicting non context statement. 3rd B1cso need correct conclusion in context containing the words in bold from a fully correct solution. For one tail need "more than 100g"	

Question Number	Scheme	Marks
3. (a)	$s_p^{\ 2} = \dfrac{12\times161+9\times48}{13+10-2} = \dfrac{2364}{21} = 112.571\ldots = 112.6\ \text{(1dp)}$	M1A1cso (2)
(b)	To test $\mathrm{H}_0 : \mu_s = \mu_a$ against $\mathrm{H}_1 : \mu_s \neq \mu_a$ (o.e.)	B1
	Test stat, $t = \pm\dfrac{195-186}{\sqrt{112.57\ldots\left(\frac{1}{10}+\frac{1}{13}\right)}} = \pm 2.016\ldots$ (awrt2.02)	M1A1
	Critical values, $t_{21} = (\pm)1.721$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1 Evidence of difference in mean **arm span** of adult male **swimmers** and adult male **athletes** or No evidence to support **Ali's claim**.	A1 (5)
(c)	To test $\mathrm{H}_0 : \sigma_s^{\ 2} = \sigma_a^{\ 2}$ against $\mathrm{H}_1 : \sigma_s^{\ 2} \neq \sigma_a^{\ 2}$	B1
	Test stat, $F_{12,9} = \dfrac{161}{48} = 3.354\ldots\ \left(\dfrac{1}{\mathrm{F}_{12,9}} = \dfrac{48}{161} = 0.2981...\right)$	M1A1
	Critical value, $F_{12,9} = 3.07\ (0.3257\ldots)$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1 Evidence of difference in **variance** of arm span of adult male swimmers and adult male athletes or the data supports **Bea's belief**	A1cso (5)
(d)	Should do test for variance first as **equal variances** is necessary assumption for *t* test for means	B1
	but is **not supported** in (c), so result in **(b) is invalid**.	B1d (2) **Total 14**
	Notes	
(a)	M1 for $\dfrac{12\times161+9\times48}{13+10-2}$ A1cso need to get awrt112.57 **or** $\dfrac{2364}{21}$ then write 112.6	
(b)	M1 $\dfrac{195-186}{\sqrt{112.6\left(\frac{1}{10}+\frac{1}{13}\right)}}$ 2nd B1 alternate method, *p* value of 0.0566 in place of critical value Final A1 requires correct conclusion in context	
(c)	1st B1 allow $\mathrm{H}_0 : \sigma_s = \sigma_a$ against $\mathrm{H}_1 : \sigma_s \neq \sigma_a$ M1 allow $\dfrac{161^2}{48^2}$ if they write the formula down Final A1 requires correct conclusion	
(d)	1st B1 equal variances is necessary assumption (may be implied by saying not equal) 2nd B1d but not supported in (c)/(variances not equal) therefore (b) result invalid	

Question Number	Scheme	Marks
4. (a)	Power function $= \mathrm{P}(\mathrm{H}_0 \text{ rejected}) = \mathrm{P}(X_1 \geq 2) + \mathrm{P}(X_1 = 1) \times \mathrm{P}(X_2 \geq 1)$ $= 1-(1-p)^6 - 6p(1-p)^5 + 6p(1-p)^5 \times (1-(1-p)^6)$ $= 1-(1-p)^6 - 6p(1-p)^5 + 6p(1-p)^5 - 6p(1-p)^{11}$ $= 1-(1-p)^6 - 6p(1-p)^{11}$	M1A1 A1cso (3)
(b)	Size of test is value of power function when $p = 0.05$ Size of test $= 1 - 0.95^6 - 6 \times 0.05 \times 0.95^{11} = 0.094268\ldots$ (awrt 0.0943)	M1A1 (2)
(c)	E[number of eggs inspected] $= 12 \times \mathrm{P}(X_1 = 1) + 6 \times \mathrm{P}(X_1 \neq 1)$ $= 12 \times 6 \times 0.1 \times 0.9^5 + 6 \times (1 - (6 \times 0.1 \times 0.9^5))$ $= 8.1257\ldots$(awrt 8.13)	M1 A1 A1 (3)
(d)	P(Type II error $\mid p = 0.1) = 1 -$ (value of power function when $p = 0.1$) P(Type II error $\mid p = 0.1) = 1 - (1 - 0.9^6 - 6 \times 0.1 \times 0.9^{11}) = 0.7197\ldots$ (awrt 0.720)	M1 A1 (2)
(e)	Prob of Type II error, accepting $p = 0.05$ when it is actually 0.1, unacceptably high, is large, therefore not a good **test**.	B1 (1)
		Total 11
	Notes	
(a)	M1 for $\mathrm{P}(X_1 \geq 2) + \mathrm{P}(X_1 = 1) \times \mathrm{P}(X_2 \geq 1)$ or $1 - \left(\mathrm{P}(X_1 = 0) + \mathrm{P}(X_1 = 1) \times \mathrm{P}(X_2 = 0)\right)$ oe or a correct line of working A1 a correct line of working before the final answer A1 fully correct solution no errors.	
(b)	M1 attempt to subst 0.05 into (a)	
(c)	M1 for $12 \times \mathrm{P}(X_1 = 1) + 6 \times \mathrm{P}(X_1 \neq 1)$ A1 $12 \times 6 \times p \times 0.9(1-p)^5 + 6 \times (1 - (6 \times p \times (1-p)^5)$	
(d)	M1 $1 - (1 - (1-p)^6 - 6 \times p \times (1-p)^{11})$	
(e)	B1 idea that the Probability of a Type II error is **too high** or the power is too low so the **test is not good/powerful or test needs changing**	

Question Number	Scheme	Marks
5 **(a)**	$\bar{x} = \frac{\sum x}{n} = \frac{1116}{9} = 124$	B1
	$s^2 = \frac{9}{8}\left(\frac{138728}{9} - 124^2\right) = 43$	
	Or	
	$s^2 = \frac{1}{8}\left(138728 - \frac{1116^2}{9}\right) = 43$	B1
		(2)
(b)	Test stat	M1A1
	$\chi^2 = \frac{8 \times 43}{25} = 13.76$	
	Critical value	
	$\chi^2 = 15.507$	B1
	Therefore not in critical region, insufficient evidence to reject H_0	
	There is evidence at the 5% level that the **company's** claim is supported	B1d
		(4)
(c)	CI given by	
	$\frac{11 \times 8.17}{21.920} < \sigma^2 < \frac{11 \times 8.17}{3.816}$	M1
	Therefore	
	$4.0999\ldots < \sigma^2 < 23.55\ldots$ awrt 4.10 and 23.6	A1
		(2)
(d)	$\sigma^2 = 25$ is not in CI which suggests **Gurdip's(his)** claim may not be true.	B1ft
		(1)
		Total 9

	Notes	
(a)	B1 124 B1 43	
(b)	M1 $\frac{8 \times \text{their } 43}{25}$ A1 awrt 13.8 B1 15.507 B1 dep on previous M1 being awarded. Allow the standard deviation of the **IQ** scores is 5 oe. Must have IQ	
(c)	M1 $\frac{11 \times 8.17}{3.816 \text{ or } 21.92}$ A1 both correct	
(d)	B1ft their interval from part(c). Gurdip's claim may not be true NB, no interval in (c) then B0	

Question Number	Scheme	Marks
6. (a)	$\mathrm{E}[A]=\frac{1}{2}(\mathrm{E}[X_1]+\mathrm{E}[X_2]+\mathrm{E}[X_3]+\mathrm{E}[Y_1]+\mathrm{E}[Y_2])=\frac{1}{2}\left(3\times\frac{\mu}{3}+2\times\frac{\mu}{2}\right)=\mu$	M1
	Therefore A is an unbiased estimator	A1
	$\mathrm{E}[B]=\frac{3\mathrm{E}[X_1]}{2}+\frac{2\mathrm{E}[Y_1]}{3}=\frac{3}{2}\times\frac{\mu}{3}+\frac{2}{3}\times\frac{\mu}{2}=\frac{5\mu}{6}$	A1
	Therefore B is biased with bias $(-)\frac{\mu}{6}$	B1ft
	$\mathrm{E}[C]=\frac{1}{3}(3\mathrm{E}[X_1]+4\mathrm{E}[Y_1])=\frac{1}{3}\left(\frac{3\mu}{3}+\frac{4\mu}{2}\right)=\mu$	
	Therefore C is an unbiased estimator	A1
		(5)
(b)	Best estimator is unbiased estimator with least variance	
	$\mathrm{Var}\,(A)=\frac{1}{4}(\mathrm{Var}\,X_1+\mathrm{Var}\,X_2+\mathrm{Var}\,X_3+\mathrm{Var}\,Y_1+\mathrm{Var}Y_2)$	M1
	$=\frac{1}{4}\left(3\times3\sigma^2+2\times\frac{\sigma^2}{2}\right)=\frac{5\sigma^2}{2}$	A1
	$\mathrm{Var}\,(C)=\frac{1}{9}(9\mathrm{Var}\,X_1+16\mathrm{Var}\,Y_1)=\frac{1}{9}\left(9\times3\sigma^2+16\times\frac{\sigma^2}{2}\right)=\frac{35\sigma^2}{9}$	A1
	Therefore A is a better estimator of μ (smaller variance)	B1dft
		(4)
(c)	$\mathrm{E}[D]=\frac{1}{k}\left(2n\times\frac{\mu}{3}+n\times\frac{\mu}{2}\right)=\mu$	M1A1
	$k=\frac{2n}{3}+\frac{n}{2}=\frac{7n}{6}$	A1
		(3)
(d)	$\mathrm{Var}\,(D)=\frac{1}{k^2}\left(2n\times3\sigma^2+n\times\frac{\sigma^2}{2}\right)=\frac{1}{k^2}\times\frac{13n\sigma^2}{2}$	M1
	$\mathrm{Var}\,(D)=\frac{36}{49n^2}\times\frac{13n\sigma^2}{2}=\frac{234\sigma^2}{49n}$	M1d A1
	Therefore $\mathrm{Var}\,D\to0$ as $n\to\infty$, therefore D is a consistent estimator	A1dd
		(4)
(e)	Want	
	$\frac{234\sigma^2}{49n}<\frac{5\sigma^2}{2}$	M1
	Therefore	
	$\frac{234}{49}\times\frac{2}{5}<n$	
	$n>1.910\ldots$	
	So minimum value is $n=2$	A1cso
		(2)
		Total 18

	Notes	
(a)	M1 for a correct method for E(A) or E(B) or E(C) A1 for each correct expectation with a correct method B1ft bias of B, condone missing – sign. Do not allow a bias of 0	
(b)	M1 Use of Var(aX) = a^2Var (X) and subst $3\sigma^2$ for Var(X) and $\frac{\sigma^2}{2}$ for Var(Y) A1 for each correct variance B1dft their variances. Dep on m1 being awarded. If no variances given then B0	
(c)	M1 attempts E(D) and puts = to μ (may be implied) A1 for E(D)	
(d)	M1 for $\frac{1}{k^2}\left(2n\times 3\sigma^2 + n\times\frac{\sigma^2}{2}\right)$ or $\frac{1}{k^2}\times\frac{13n\sigma^2}{2}$ M1d for subst in k A1 Correct Var (D) A1dd Need correct reason for being a consistent estimator dep on previous method marks being awarded	
(e)	M1 for forming an inequality with their Var(D) < their best estimator leading to n	

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom